



An Ecological Model of Place-based Deterrence

P. Jeffrey Brantingham¹ 

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Abstract

Purpose The central premise behind place-based policing is that an intervention narrowly targeted to a location is able to suppress crime for some period of time. The crime-free survival time in a place ends with prolonged exposure to police action, known as initial deterrence decay, or after police have left, known as residual deterrence decay. The purpose of the present work is to understand the origin and character of deterrence decay at an aggregate spatial scale.

Methods Deviating from previous efforts that explain deterrence decay based on the psychology of offender decision-making, the present work borrows ideas from theoretical ecology to model place-based deterrence as a form of competition between police and offenders over space. Deterrence decay emerges as a byproduct of this competition.

Results When measured on an aggregate spatial scale, the model suggests that the waiting time to the emergence of crime and disorder from the onset of place-based policing actions should be gamma-like in distribution. The waiting time from the end of a place-based police action should be exponentially distributed.

Conclusion If the model is a reasonable approximation of reality, then attempts to schedule place-based maintenance visits to counteract deterrence decay may be of limited value.

Keywords Hot spot policing · Ecological competition · Ordinary differential equations · Survival analysis

Introduction

Studies dating back decades show that place-based policing yields measurable reductions in crime (Braga 2001; Braga et al. 2014; Braga and Weisburd 2022). Crime reductions are observed following spatially targeted pedestrian stops (Petersen et al. 2023), hotspot patrols (Sherman 1990; Braga et al. 2024), and more substantial forms of place-based

✉ P. Jeffrey Brantingham
branting@ucla.edu

¹ Department of Anthropology, University of California Los Angeles, 375 Portola Plaza, Los Angeles, CA 90095, USA

environmental modification (MacDonald et al. 2024). However, the effects of place-based police are strictly localized. In space, place-based interventions typically suppress crime in the immediate vicinity of the action, as well as nearby areas that appear to benefit indirectly from crime control measures taken ‘around the corner’ (Clarke and Weisburd 1994; Weisburd et al. 2006, 2012). In time, crime reduction begins almost immediately after the onset of a place-based intervention and may persist for some time after the visible elements of an intervention stop (Koper 1995; Sherman 1990). The widely appreciated policy implication is that place-based policing needs to be precisely targeted and periodically reinforced (Sherman 1990; Dau et al. 2023).

The apparent need to reinforce place-based policing interventions is consistent with a theory of deterrence focused on offender decision-making processes (Koper 1995; Sherman and Weisburd 1995; Nagin 1998; Loughran et al. 2011). Specifically, hot spot policing is thought to alter local offenders’ perceptions of the risk of getting caught. The *initial deterrence* effect arises while there is visible police action, including physical presence, enforced sanctions, and continued publicity about the action (Sherman 1990). Presumably, offenders recognize that committing a crime in plain sight of the police comes with a high probability of being caught and punished (Nagin 1998; Loughran et al. 2011). The *residual deterrence* effect, by contrast, is thought to arise after the visible elements of police action have ceased. The theory is that, although police are not immediately visible, offenders monitoring a targeted place are uncertain about whether patrols will soon return. This ambiguity discourages offenders from committing crime (Koper 1995; Nagin et al. 2015; Loughran et al. 2011). However, initial and residual deterrence effects are not persistent. Limited police resources mean that police must frequently move to other places (Sherman 1990). Theory holds that offenders gradually realize that police are unlikely to return, reducing the ambiguity surrounding the chance of getting caught. The shift in perception allows offenders to return to business as usual, ending the period during which the place remained crime-free. Police therefore need to return to that location if they wish to restore deterrence.

Several important theoretical and practical questions surround the time it takes for crime and disorder to re-emerge given a place-based policing intervention. Sherman (1990) identified *initial deterrence decay* as a process by which offenders learn—by word of mouth or trial-and-error experimentation—that they overestimated the risk of getting caught despite the continued presence of police. Thus, crime and disorder might re-emerge even before a place-based intervention has ended (Sorg et al. 2013). Sherman also identified *residual deterrence decay* as a similar learning process that operates only after police have visibly left a place. If crime and disorder do not re-emerge while police are present, it is likely to do so after police have left the scene. In both cases, deterrence decay implies that the probability that a place remains crime-free decreases over time. As a theoretical matter, the specific functional form that describes deterrence decay may tell us something about the mechanisms underlying how offenders select crime sites and evaluate risk. As a practical matter, knowing the functional form of deterrence decay may allow police to optimally plan the duration and timing of place-based patrols to have a lasting impact on crime (Williams and Coupe 2017).

Addressing both issues, Koper (1995), using data from the Minneapolis Preventive Patrol Experiment (Sherman and Weisburd 1995), suggested that deterrence decay could be slowed (up to a point) by longer place-based patrols. The Minneapolis experiment randomly

assigned 29 trained observers across 100 candidate hotspots for 70-minute observation windows. Observers recorded the start and end times of police activity and street disorder. Koper's event history analysis of the experimental data focused on the waiting time between the end of observed policing events and the start of new disorder events, subject to several important data constraints detailed in the study (see Koper 1995:660-61). His analysis suggested that patrol stops lasting 11-15 minutes had longer survival times (measured in minutes) until the next observed disorder event compared to short "drive-by" patrols (Fig. 1A). The benefits peaked at approximately 15 minutes of patrol, after which there were no clear gains in survival time (Fig. 1B). This core observation about 15-minute place-based patrols is widely known among academics and police practitioners as the "Koper Curve" (Sherman and Eck 2002).

Subsequent studies have supported the hypothesis that targeted patrols lasting approximately 15 minutes outperform call-driven patrol practice and shorter-duration hot spot patrols in reducing crime. Telep et al. (2014), for example, conducted a randomized controlled trial comparing the outcomes in street segments subject to 12-16 minutes of directed patrol every two hours (treatment) with segments subject to incidental routine patrol (control). Treated street segments received twice as much patrol time as the control segments during the 90-day experiment and saw a significant reduction in both calls-for-service and Part I crimes. Williams and Coupe (2017) implemented a similar random block experimental design comparing treatment hotspots randomly assigned to receive three 15-minute patrols on some days and nine 5-minute patrols on others. Although patrol dosage proved challenging to measure, evidence suggests that fewer, longer patrols in targeted hotspots had a larger effect on crime than more frequent, shorter patrols. However, neither of the above studies directly investigated the relationship between patrol duration and initial or residual deterrence (Telep et al. 2014; Williams and Coupe 2017).

In contrast, Barnes et al. (2020) used a crossover design that randomized treatment assignments by day across 15 hotspot locations. Treatment locations received one or more targeted police efforts each lasting 5-30 minutes, while control locations received business-as-usual patrol. The study did not try to measure the deterrent effects of patrol bouts of different lengths, but did examine residual deterrence quantified as the prevalence of offending in control hotspots as a function of days since a hotspot was last assigned to treatment. They found that 11.4% of control hotspots recorded at least one offense

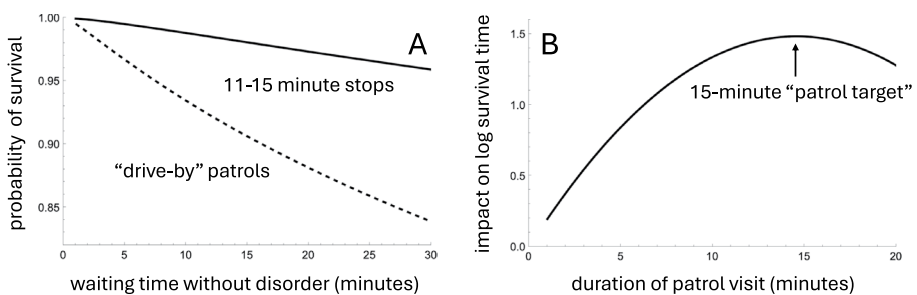


Fig. 1 Key features of the Koper Curve. A. The survival probability that a hot spot remains crime free over follow-up times up to 30 minutes for short "drive-by" patrols and 11-15 minute patrol visits. B. The impact on survival time for patrol visits of increasing duration, peaking at just under 15 minutes. Figures recreated from parameters reported in Koper (1995)

per day on each of the first four days following the last targeted patrol, similar to the prevalence under treatment (Barnes et al. 2020). Offending prevalence jumped to 13.9% after five days. Although the effect was not significant, the authors suggest that the pattern reflects residual deterrence lasting several days after a targeted patrol before “collapsing.” Replicating key parts of Barnes et al.’s design, Bland et al. (2021) failed to find a multi-day residual deterrent effect, however. Sorg et al. (2013) considered the problem of initial and residual deterrence on a larger spatial scale using data from the Philadelphia Foot Patrol Experiment. Foot patrols were randomly assigned to 60 treatment beats that contained an average of 23.9 street segments each. Beats were patrolled by two-person teams in back-to-back 8-hour shifts, five days a week, over two phases of deployment lasting 22 and 12 weeks, respectively. Violent crime declined in treatment beats relative to matched controls during both phases of deployment. The observed deterrence effects remained stable during the second phase of deployment but decayed during the first phase of deployment, which was 10 weeks longer. No residual deterrence effects were observed in treatment beats after either deployment phase.

Although each of the above studies tests some aspect of the Koper Curve, they do not offer uniform support for place-based residual deterrence effects. They are more consistent in their support for initial deterrence effects. With the possible exception of Barnes et al. (2020), none of the studies suggest a particular functional form for deterrence decay that would support or refute the model proposed by Koper (1995). Clearly, further observational and experimental studies of place-based deterrence are still warranted (Ariel et al. 2020). Here, however, I take a purely theoretical approach and use a formal model to examine some of the mechanisms that might underlie place-based deterrence and deterrence decay. The central question is: Can one derive from theory (rather than estimate statistically) what deterrence decay should look like? To make progress on this question, I turn to ideas that originated in theoretical ecology (Levin 1974; Nee and May 1992; Tilman 1994) and the study of stochastic dynamical systems, particularly disease epidemics (Brauer et al. 2019; Tang et al. 2020). The model developed here tracks the proportion of sites or locations in an environment where offending is deterred p_1 and the proportion of sites that are suitable for crime p_2 . These are treated as mutually exclusive but transitory states requiring periodic renewal or maintenance. The interaction between environmental states is modeled using coupled ordinary differential equations, capturing in mathematical form the competition between places that are favorable and unfavorable for crime. The model provides a way to formally specify the rate at which place-based initial deterrence ends λ_{10} and, separately, the rate at which suitable crime sites are discovered λ_{02} . I argue that λ_{02} is closely related to residual deterrence. Depending on how these two rates are combined, we arrive at different theoretical expectations about the survival time for a place to remain crime-free. Neither of the theoretical forms looks much like the pattern of residual deterrence estimated statistically by Koper (1995).

This last observation is not intended as a criticism of Koper’s foundational work, and the following analyses should not be misconstrued as a test of the Koper Curve. Rather, the discrepancies in observations stem from differences in methods and modeling assumptions. Methodologically, the present paper takes a “first-principles approach” to the relationship between policing and place-based deterrence using a range of mathematical modeling tools. This is substantially different from the approach taken by Koper who started with nearly 17, 000 street-level observations of active policing and disorder

events and then proceeded to estimate deterrence effects with event-history statistical models (Koper 1995). The model introduced below is concerned with general questions of mechanisms and dynamics, not with the experimental design or sampling frames needed to estimate deterrence effects from data. The present model also makes important simplifying assumptions that are more about spatial processes than about the psychology of deterrence (Nagin 1998; Loughran et al. 2011; Nagin et al. 2015). Here, the interactions between police and offenders are considered only at an aggregate scale and concern primarily the processes by which open space is explored and exploited by both groups. A central assumption is that it takes time (and effort) for offenders to find and exploit places suitable for crime, and that residual deterrence might emerge from this spatial process. The internal cognitive processes in which offenders weigh the risk of getting caught play almost no role in what appears below. The assumptions are reasonable and plausible in this context, and the modeled results are not widely out of line with what is known empirically about patterns of place-based deterrence and deterrence decay. I do not claim that this ecologically-inspired model replaces the individual-scale microeconomic or cognitive models of place-based deterrence referenced above (Nagin 1998; Loughran et al. 2011). Rather, I see aggregate- and individual-scale models as complementary. Future work will be needed to understand how they align.

The remainder of this paper is structured as follows. In Section “[Deterrence and Crime as Place-based Competition](#)”, I describe the elements of the model and illustrate the basic dynamics of the idealized system. Section “[Place-based Turnover](#)” turns to an analysis of the model, focusing on its interpretation in terms of landscape-scale equilibrium processes of place-based deterrence and offender crime site selection. The focus is on the landscape-scale turnover of place-based deterrence and conditions favorable to crime given the model system. Section “[Deriving Deterrence Decay](#)” turns to the derivation of expressions for the crime-free survival time of a place due to initial deterrence, residual deterrence, and total deterrence (the combination of initial and residual deterrence). Section “[Discussion](#)” discusses the implications of the model for both theory and practice and suggests possible next steps. Supplementary information includes both *Mathematica* and *Python* code sufficient to replicate the results and useful for building additional experiments.

Deterrence and Crime as Place-based Competition

In this section, I develop a spatially implicit model that treats place-based policing, on the one hand, and offender discovery and exploitation of suitable crime sites, on the other, as a form of competition over space (Kohfeld and Sprague 1990). The model is inspired by a series of articles originating in theoretical ecology (Levin 1974; Nee and May 1992; Tilman 1994). These papers explore the problem of how inferior competitors manage to survive in the face of superior competitors that are able to not only colonize and hold open space but also displace inferior competitors wherever they are encountered. In general, these conditions should lead to competitive exclusion, but in practice inferior competitors not only survive but thrive in the face of seeming absolute competitive advantage. A solution to this paradox, explored by Tilman (1994), hinges on how inferior competitors manage to exploit the gaps in the spatial footprint of superior competitors. Here, the same

mechanism is assumed to allow suitable crime sites to persist in the face of effective place-based policing. The model is then used to explore the temporal patterns generated through turnover in places with initial deterrence and sites that are suitable for crime.

Consider as a starting point an abstract environment consisting of N discrete places that might host either police or offender activity. The places I have in mind are the micro-geographic settings common in studies of place-based policing such as street segments or sub-neighborhood-sized crime hot spots (Weisburd 2008; Hipp and Williams 2020; Mohler et al. 2015). I assume that police activity in any one of these places establishes conditions that are locally unfavorable to crime and disorder. The activities that I have in mind are those that can be accomplished in relatively short visits as part of a routine police patrol (e.g., 15 minutes on average) (Koper 1995; Telep et al. 2014). The model could also be framed in terms of other place-based tactics, including problem solving, third-party policing efforts, or the deployment of surveillance technologies (Groff et al. 2015; MacDonald et al. 2024), although possibly with different implied time scales. In contrast, offender activity in any one of these places establishes conditions that are favorable or suitable for crime. Offender activities that might produce conditions locally favorable or unfavorable to crime could include learning about and exploiting static environmental characteristics such as vulnerable targets (Nee et al. 2019) and unintentional (or intentional) actions that change local situational conditions (Keizer et al. 2008; Odling-Smee et al. 1996).

I assume that the conditions unfavorable to crime are competitively superior to the conditions favorable to crime. For example, if police activity begins in a location currently suitable for crime, then conditions favorable to crime are assumed to be completely suppressed. The location immediately becomes unsuitable for crime. This is an extreme assumption that is likely violated in real ecological settings. Police may fail to observe crime occurring in their midst or may observe crime but exercise discretion and choose not to enforce the law. Nevertheless, the extreme assumption provides a useful baseline for what a world with absolute local crime suppression might look like. Similarly, if a location is currently unfavorable for crime because of active initial deterrence, I also assume that offenders are not able to unilaterally override those unfavorable conditions. Initial deterrence must end before a location can again become favorable for crime. This is also an extreme but useful assumption. In reality, offenders may learn that the risk of apprehension is quite low even when the police are nearby (Sherman 1990). In contrast, the absence of initial deterrence in a place allows or, more accurately, does not block the emergence of suitable crime sites. Rather, because police are absent in such settings, I assume that offenders can discover, cultivate, and exploit criminal opportunities there.

In the above formulation, initial deterrence and suitability for crime are mutually exclusive spatial states. That is, at any one moment in time, a site can exist only in one of three states: (1) initial deterrence; (2) suitable for crime; or (3) neither initial deterrence nor suitable for crime (i.e., empty or neutral space). Let $P_1(t)$ be the number of sites experiencing initial deterrence at time t and $P_2(t)$ the number of sites suitable for crime. Thus, $p_1 = P_1(t)/N$ is the proportion of sites in the environment at time t with active deterrence and $p_2 = P_2(t)/N$ the proportion of sites suitable for crime. The proportion of sites at time t in neither of these two states is $1 - p_1 - p_2$. The core question of interest concerns how these proportions evolve over time.

Here, I introduce a type of epidemiological process that couples change in the proportion of space suitable for crime to change in the proportion of space given over to initial deterrence. Conceptually, given unlimited personnel, equipment, and communication resources, police would be able to establish and maintain place-based initial deterrence throughout the environment (i.e., $p_1^* = 1$ at equilibrium). Conversely, in the complete absence of policing resources and the presence of an unlimited and well-resourced offender population, all space would be given over to crime (i.e., $p_2^* = 1$ at equilibrium). However, realistically, resource constraints limit the ability of police to establish and maintain active deterrence everywhere (Sherman 1990), while offender population constraints limit their ability to convert all open locations into sites suitable for crime were they free to do so (Rossmo and Routledge 1990; Chainey and Lazarus 2021).

To capture these ideas more formally, let c_1 be the intrinsic rate at which police establish initial deterrence in places throughout the environment and m_1 the intrinsic rate at which initial deterrence ends in places where it was previously established. Similarly, let c_2 be the intrinsic rate at which offenders discover and establish places as suitable for crime and m_2 the intrinsic rate at which those locations cease being suitable.¹ I assume that each of the parameters is constant because of resource constraints.

Given the above terms, one can write the following spatially implicit² model consisting of coupled Lotka-Volterra-type ordinary differential equations (Levin 1974; Nee and May 1992; Tilman 1994):

$$\frac{dp_1}{dt} = c_1 p_1 (1 - p_1) - m_1 p_1 \quad (1)$$

$$\frac{dp_2}{dt} = c_2 p_2 (1 - p_1 - p_2) - m_2 p_2 - c_1 p_1 p_2 \quad (2)$$

Equation 1 describes the instantaneous rate of change in the proportion of places p_1 with initial deterrence. Equation 2 describes the instantaneous rate of change in the proportion of places p_2 that are suitable for crime. Solving the coupled equations for different parameter values produces outcomes where initial deterrence p_1 or suitable crime sites p_2 dominate everywhere, or there is a linearly stable equilibrium with some portion of space with active initial deterrence, some proportion of space that is suitable for crime, and some proportion of space that is in neither state (Fig. 2) (Tilman 1994). The latter case will be the focus in what follows. *Mathematica* code is provided as supplementary information to replicate the behavior of the differential equations. Supplemental *Python* code leverages Euler's method to simulate the system using discrete equations of the form $p_i(t + \Delta t) = p_i(t) + f(p_i, t)\Delta t$, where $p_i(t + 1)$ is the proportion of space in state i in the next time step, $f(p_i, t)$ is the right-hand side of Eqs. 1 or 2, evaluated using the value of p_i in the current time step t , and Δt is the time step size.

¹These terms are related to an SIR-type epidemiological model with a reproductive number for the growth of unfavorable or favorable places for crime across the environment given by $R0 = c/m$.

²The model is spatially implicit because it tracks global proportions rather than explicit spatial locations at which conditions are unfavorable or favorable for crime. Many different spatial arrangements can produce the same proportions and thus potentially complex local spatial processes are held in the background.

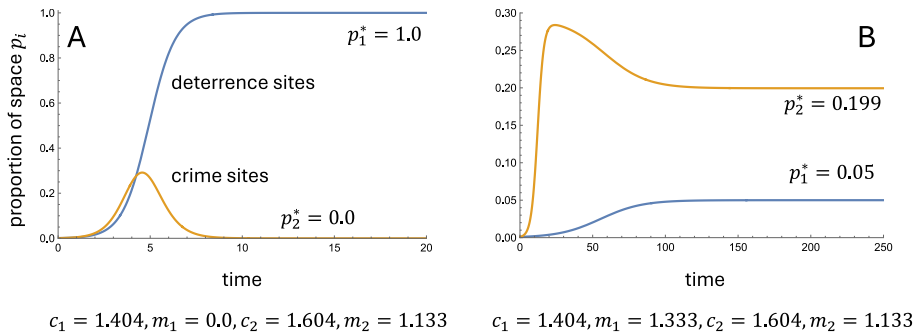


Fig. 2 Evolution of the proportion of space with active deterrence p_1 (blue) and suitable for crime p_2 (orange). (A) A ‘zero crime police state’ arises when there is no limit to the growth of initial deterrence $m_1 = 0$. (B) An ‘abundant crime world’ world that includes all of the terms in Eqs. 1 and 2. Parameters are listed below the figures and are chosen so that time units can be interpreted as minutes. Initial conditions are $p_1(0) = p_2(0) = 0.001$

The individual terms in Eqs. 1 and 2 have very specific interpretations. The first term $c_1 p_1(1 - p_1)$ in Eq. 1 indicates that the growth of place-based deterrence is density dependent. The proportion of the environment with active deterrence grows rapidly from low levels as more resources are put into the field. This suggests that police find it progressively easier to establish place-based deterrence as they occupy more space. Density dependence could arise if patrol units already established in the p_1 proportion of space learn something about current conditions that allow the next set of patrol units entering the system to find where they need to go more quickly (Famega et al. 2005). Appendix A shows that a model with no density dependent growth produces qualitatively similar results. Growth is not unbounded, however. Two processes limit the growth of deterrence across space. First, growth is slowed by the reduction of open space; i.e., the proportion of empty space $(1 - p_1)$ tends towards zero as the proportion of space with initial deterrence p_1 tends towards one. In the absence of other processes, the product $p_1(1 - p_1)$ leads initial deterrence to grow logistically to eventually fill the entire environment. This is a ‘zero-crime police state’ (Fig. 2A).

The second limit on the growth of initial deterrence starts with the assumption that police activity in a place cannot be sustained indefinitely. This assumption is inherent in the distinction between initial and residual deterrence made by Sherman (1990). Thus, the second term in Eq. 1 $-m_1 p_1$ forces decay of place-based initial deterrence. However, it is critical to note that decay operates in the global proportion of sites with active deterrence p_1 , not in the deterrence observed at any one discrete site. Deterrence decay in Eq. 1 is proportional; that is, as p_1 approaches one, it becomes harder to maintain deterrence throughout space, increasing the rate at which deterrence decays to a maximum of m_1 . Intuitively, we can think of global deterrence decay as representing the rising costs of having to police more and more space with limited patrol resources.

Note that the growth of deterrence is not impeded by the presence of places suitable for crime. Specifically, p_2 does not appear explicitly anywhere in Eq. 1. Thus, deterrence spreads equally well in open or empty spaces as in places currently suitable for crime. Section “Place-based Turnover” below examines more closely the fraction of places in the landscape where deterrence is established suppressing local conditions that are suitable for crime.

The first and second terms in Eq. 1 play off one another such that if $m_1 > 0$ spatial gaps in active deterrence are guaranteed to form (Tilman 1994). In Fig. 2B, police establish active deterrence over 5% of space at equilibrium, leaving 95% of space potentially open for criminal offending. Different parameter values for c_1 and m_1 will fill space with initial deterrence to different degrees. The parameter values in Fig. 2 B are chosen here to be consistent with key parts of Koper's (1995) study. As discussed in more detail below, m_1 is chosen so that the mean waiting time to the end of initial deterrence is 15 time units (e.g., minutes) and $c_1 = (1 - p_1^*)/m_1$ assuming $p_1^* = 0.05$. Thus, the parameter values produce initial deterrence at a low level across space, as is typical of most urban settings under normal policing conditions.

In this idealized world, offending behavior may fill some or all of the spatial gap left by global deterrence decay. The first term in Eq. 2 $c_2 p_2 (1 - p_1 - p_2)$ captures this dynamic precisely. Note that the proportion of space discovered and made suitable for offending tends to grow at a rate proportional to $c_2 p_2$. This density dependence suggests that it becomes easier to discover and establish places as suitable for crime as more of the environment is given over to these conditions. This assumption is consistent with both the idea that 'broken windows' may encourage the spreading of disorder and that crime itself is spatially contagious via mechanisms such as near-repeat victimization (Keizer et al. 2008; Townsley et al. 2003). However, the spread of suitable places for crime is limited by the proportion of space that is under active initial deterrence *and* the proportion already given over to crime $(1 - p_1 - p_2)$. If this were all that mattered, places suitable for crime would grow to fill all empty space without active deterrence. In fact, this propensity is evident in Fig. 2B where, early in the time series, the proportion of space suitable for crime overshoots the eventual equilibrium value before then decreasing. This happens because deterrence is rare at the outset, allowing crime site discovery to proceed at or near its maximum rate. As with place-based deterrence, however, I assume that the conditions favorable to crime do not remain continually active. That is, suitable crime sites can fall out of favor even without the intervention of police and even though crime opportunities remain available. Intuitively, given a finite population of offenders exploiting a finite set of locations, it is reasonable to assume that they cannot continually monitor or keep track of all of the places that might be suitable for crime, nor can they maintain indefinitely the local social networks or environmental cues that make a place favorable to crime. Thus, $-m_2 p_2$ describes the proportionally dependent rate at which crime suitability naturally decays. As p_2 grows, it becomes proportionally harder to maintain favorable conditions for crime across space. Again, this decay in conditions favorable to crime is recognized on an aggregate scale across space, not at individual sites.

The third term in Eq. 2 $-c_1 p_1 p_2$ describes the asymmetric relationship between deterrence and crime. Specifically, active deterrence 'suppresses' suitable crime places, which is modeled as a negative rate of change in the proportion of sites suitable for crime. Crime suppression here is tied to the intrinsic rate at which deterrence grows c_1 but varies according to the interactions between the proportion of space with active initial deterrence and the proportion of space with suitable crime sites. The product $c_1 p_1 p_2$ can be conceptualized in probabilistic terms: p_1 is the probability that initial deterrence is extended to a new random location, p_2 is the probability that a random location is presently suitable for crime, the product is the joint probability that initial deterrence randomly targets a suitable crime site, and c_1 is the intrinsic rate at which these interactions can happen.³

³In demographic and epidemiological models $p_1 p_2$ is interpreted as the random encounter rate between individuals of type 1 and 2 in a well-mixed population.

Place-based suppression removes once suitable crime sites. However, the model assumes that there is no interference in the other direction; i.e., there is no complementary term $-c_2 p_2 p_1$ in Eq. 1 that would capture the idea that crime spreads to suppress active initial deterrence. In Eq. 2, the combined result of spatial packing and crime suppression is that only a fraction of the space not under active deterrence is converted to sites suitable for crime. In Fig. 2B, for example, suitable crime sites at equilibrium take up only 20.0% of space, leaving 75.0% of the environment open or free of both active deterrence and crime. The situation represented in Fig. 2B is a ‘crime abundant world’ where places suitable for crime are more abundant than places with initial deterrence, although most places are safe and ‘unpoliced.’

Although not the focus of the present work, it is still important to identify the parameter regimes that allow at least some portion of space to be suitable for crime at equilibrium (i.e., $p_2^* > 0$) while deterrence is also present (i.e., $p_1^* > 0$). For Eqs. 1 and 2, suitable crime sites exist in the face of initial deterrence as long as (Tilman 1994):

$$c_1 > m_1 \quad (3)$$

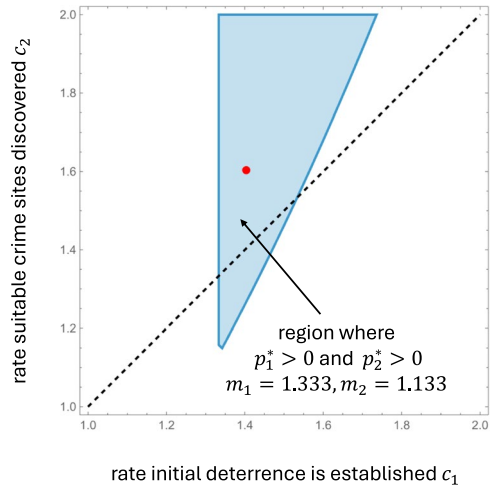
$$c_2 > c_1 \left(\frac{p_1^*}{1 - p_1^*} + \frac{m_2}{m_1} \right) \quad (4)$$

Equation 3 provides the necessary condition for there to be some amount of initial deterrence active across space at equilibrium. Equation 4 then holds (with a narrow window for exceptions) that the persistence of favorable conditions for crime requires the intrinsic rate at which crime sites are discovered c_2 to be greater than the intrinsic rate at which initial deterrence is established c_1 (Tilman 1994).⁴ For example, in Fig. 3 the shaded region shows values of c_1 and c_2 that can lead to $p_1^* > 0$ and $p_2^* > 0$ given $m_1 = 1.333$ and $m_2 = 1.1333$ as in Fig. 2B. The actual values of c_1 and c_2 used in Fig. 2B are shown as a red point. Intuitively, this means that offenders need to be able to find and exploit open space at an intrinsic rate faster than police are able establish deterrence. Although an interesting result, further exploration of the parameter space is not pursued here.

Finally, it is worth briefly describing a few subtle implications of the equilibrium conditions of the model. Focusing on the system after around 150 time units in Fig. 2B, it is clear that the proportion of space that remains suitable for crime does not change despite active place-based deterrence. There are two points to be made about this equilibrium state. First, a reduction in the global proportion of space suitable for crime would require changes in the intrinsic rates that drive the model. More police resources could be brought to bear that would increase the rate at which initial deterrence is established in new places (i.e., larger c_1) or slow the rate at which initial deterrence globally decays (i.e., smaller m_1). Alternatively, interventions (other than place-based deterrence) could decrease the intrinsic rate at which offenders discover and establish conditions favorable for crime (i.e., smaller c_2) or increase the rate at which such conditions decay (i.e., larger m_2). If such were to occur, then we should expect to

⁴The conditions are obviously more nuanced than this simple conclusion. Tilman (1994) shows that inferior competitors who can hold places longer than superior ones (i.e., $m_2 < m_1$) can also produce $p_2^* > 0$ but these conditions are more restrictive.

Fig. 3 The region of c_1, c_2 parameter space that supports a non-zero equilibrium proportion of initial deterrence $p_1^* > 0$ and suitable crime sites $p_2^* > 0$ for the inequalities given by Eqs. 3 and 4. The position of c_1, c_2 from Fig. 2B is shown as a red point. With some exceptions, c_2 generally must be greater than c_1 for suitable crime sites to exist given initial deterrence active in the environment (Tilman 1994)



see equilibrium proportions change accordingly. Absent such changes, the equilibrium volume of policing and crime would not change. However, this interpretation assumes a constant world. In real-world systems, we would expect the parameter values to change naturally over time such that the proportion of space under initial deterrence and suitable for crime would not appear stationary.

Second, the equilibrium conditions create the false impression that the system is static. Recall that this is a spatially implicit model that tracks the proportion of space with initial deterrence p_1 and suitable for crime p_2 on an aggregate scale. Although the aggregate quantities do not change at equilibrium, the specific places with initial deterrence, currently suitable for crime, or in an open state with conditions neither favorable nor unfavorable to crime, are constantly turning over. There is a hidden dynamic landscape of competition between deterrence and crime that generates the aggregate pattern (Kohfeld and Sprague 1990). The next two sections borrow ideas from mathematical epidemiology to examine the types of landscape turnover and the character of deterrence decay implied by the model (Southall et al. 2023).

Place-based Turnover

Equations 1 and 2 model the rate of change in the proportion of sites with initial deterrence p_1 and the proportion suitable for crime p_2 . While these quantities appear stable at equilibrium, below the surface specific sites are constantly changing state. For example, police may leave one site, ending the period of initial deterrence there, but immediately establish deterrence somewhere else. At equilibrium, the proportion p_1^* does not change but the spatial arrangement of sites with initial deterrence does. Similarly, offenders can discover or create favorable conditions for crime in one place while abandoning another at the same time, altering the spatial arrangement but not the equilibrium proportion p_2^* . The same dynamic holds for empty or neutral locations. This dynamic reading of the model means that we can interpret the individual terms in each equation as transition rates for the hidden process of places moving between different states (Southall et al. 2023; Gillespie 1977).

An expanded version of Eq. 1 (with Eq. 2 repeated for completeness) makes the range of possible place-based state transitions clear:

$$\frac{dp_1}{dt} = c_1 p_1 (1 - p_1 - p_2) + c_1 p_1 p_2 - m_1 p_1 \quad (5)$$

$$\frac{dp_2}{dt} = c_2 p_2 (1 - p_1 - p_2) - m_2 p_2 - c_1 p_1 p_2 \quad (6)$$

Noting that $c_1 p_1 p_2$ now appears in both Eqs. 5 and 6 but with opposite signs, the individual terms correspond to five distinct state transitions ongoing across space at equilibrium:

- $\lambda_{01} = c_1 p_1^* (1 - p_1^* - p_2^*)$, the rate at which initial deterrence is established in open or empty places.
- $\lambda_{10} = m_1 p_1^*$, the rate at which initial deterrence ends.
- $\lambda_{02} = c_2 p_2^* (1 - p_1^* - p_2^*)$, the rate at which suitable crime sites are discovered in empty or open places.
- $\lambda_{20} = m_2 p_2^*$, the rate at which suitable crime sites naturally fall out of use.
- $\lambda_{21} = c_1 p_1^* p_2^*$, the rate at which initial deterrence suppresses suitable crime sites.

The subscripts on λ_{ij} make explicit that the transition is from state i to state j and $i, j = 0$ designates an open or empty place. At equilibrium, $\lambda_{01} + \lambda_{21} = \lambda_{10}$ and $\lambda_{02} = \lambda_{20} + \lambda_{21}$. That is, transitions adding a new state somewhere in the landscape are balanced by transitions that subtract that state from somewhere else, suggesting that the transition rates are constant and ensuring that the equilibrium condition $dp_1/dt = dp_2/dt = 0$ is satisfied.

Constant transitions rates at equilibrium imply that the system is well mixed and also that the underlying discrete transition events are independent and identically distributed within each transition type, which are conceptually similar to the “reaction channels” of Gillespie (2001). Under these conditions, each transition rate defines a stationary Poisson process operating at the landscape scale. The expected number of Poisson state transitions from i to j in some interval of time Δt is then $E[N_{ij}] = \lambda_{ij} \Delta t$. Although more complex underlying processes could be assumed, including Lévi jump processes (Chaturapruek et al. 2013) or self-exciting point processes (Mohler et al. 2011), a Poisson process offers a relatively simple starting point to explore place-based turnover and the character of deterrence decay.

Place-based turnover refers to the sequence of transitions that locations across space go through. Here, I confine the analysis to a sequence of exactly two state transitions from initial deterrence to empty (or neutral) space and then from empty to suitable for crime. To model the sequence, I use a compartmental model derived from Eqs. 5 and 6 (Fig. 4A). The compartments in Fig. 4A are the different possible states and the arrows show the direction of transitions between states. At equilibrium, we know the proportion of space represented by each compartment (labeled p_1^* , etc.), although they are not drawn to scale (see Fig. 2B for the proportions). We also know the proportion of each compartment that originated from some immediately prior state. Specifically, at equilibrium, all of the locations that comprise empty space at any one moment arose either from locations that were previously

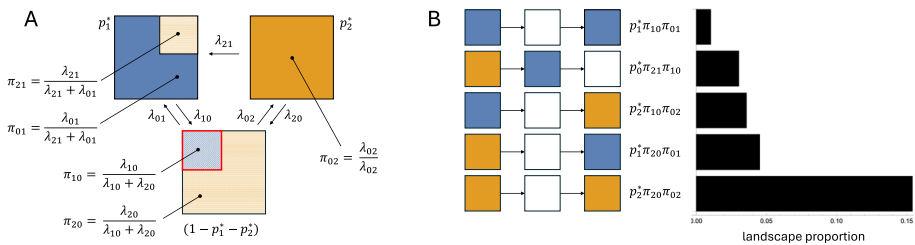


Fig. 4 A. Compartment model corresponding to Eqs. 5 and 6 showing the proportions within each compartment originating from another compartment. Region region in red is the proportion of empty space that previously was subject to initial deterrence. B. Equilibrium proportions of the environment that follows each unique two-state transition sequence. Proportions in B are based on model parameters from Fig. 2B

subject to initial deterrence or previously suitable for crime; there are only two arrows pointing towards the empty space compartment. The proportion of empty space originating from initial deterrence (light blue, outlined in red) is given by $\pi_{10}^* = \lambda_{10} / (\lambda_{10} + \lambda_{20})$. The proportion of empty space originating from previously suitable crime sites (light orange) is the complement given by $\pi_{20}^* = \lambda_{20} / (\lambda_{10} + \lambda_{20})$. For the parameter values used in Fig. 2B, a proportion $\pi_{10}^* = 0.228$ of empty space was subject to initial deterrence in the immediately prior state. For completeness, the proportion of empty space that was suitable for crime in the immediate prior state is $\pi_{20}^* = 0.772$.

Turning to the landscape scale, the proportion of the environment that is currently empty but previously subject to initial deterrence is given by $p_{10}^* = (1 - p_1^* - p_2^*) * \pi_{10}^*$. That is, p_{ij}^* is used to denote the equilibrium proportion of all space that followed a transition from state i to state j . For the parameter values used in Fig. 2B, around 17% of space at any given moment followed the state sequence from deterrence to empty. The landscape-scale proportions for other combinations of current and immediately prior state are shown in Fig. 4A.

Extending the above computations to sequences of three states, with two intervening state transitions, is relatively straightforward (Fig. 4B). For example, the proportion of space currently suitable for crime that transitioned from empty space and, prior to that, space subject to initial deterrence, is given by $\pi_{10} \pi_{02}$. This computation concerns only the compartment for space suitable for crime. At the landscape scale, the proportion of the total environment that followed this sequence is given by $p_2^* \pi_{10} \pi_{02}$. For the parameters in Fig. 2B, around 3.1% of the landscape, measured at a single moment, followed this sequence, the second lowest among all sequences. The most common sequence involves sites suitable for crime that go quiescent and then return to crime suitability. Around 15.4% of the landscape, measured at a single moment, followed this sequence. The rarest turnover sequence involves initial deterrence separated by an empty or neutral period and a second period of initial deterrence. Around 1.0% of the landscape, measured at a single moment, followed this sequence. Intuitively, the relative prevalence of the different sequences makes sense given the proportions of initial deterrence and crime site suitability at equilibrium. Appendix B illustrates the entire branching structure for all possible three-state sequences and shows that all sites measured instantaneously must have followed one of those sequences.

Deriving Deterrence Decay

The end of initial deterrence followed by the re-emergence of crime suitability is exactly the sequence of events of interest in studies of place-based deterrence. The model so far provides a way to understand the proportion of the landscape experiencing that sequence of events (Fig. 4). However, we are interested in more than just whether this sequence occurs and how common it is. We also want to understand the temporal dynamics of the sequence, particularly the form of deterrence decay or how long it takes for crime to re-emerge once initial deterrence has ended.

Two of the rates defined above are core to understanding place-based deterrence decay. Recall that crime can only occur in a place with active deterrence after that deterrence ends. Extending the logic, assume that we focus on the proportion of space where we observe the arrival of a patrol unit. These are the locations where we know that initial deterrence has started. We are not strictly concerned with λ_{01} , the rate at which active deterrence is established, nor λ_{21} , the rate at which suitable crime sites are removed through suppression. These rates are important only insofar as they generate the observational sample for sites with initial deterrence. We are also not concerned with λ_{20} , the rate at which suitable crime sites naturally fall out of use, since place-based deterrence by definition is not involved in such a process.

Given the subset of sites with initial deterrence, we are then interested in the probability that initial deterrence ends after τ_1 time units. Since the rate λ_{10} is constant at equilibrium, we can model the end of initial deterrence as a stationary Poisson process. The probability density function for τ_1 is then $f(\tau_1) = \lambda_{10}e^{-\lambda_{10}\tau_1}$. Although police patrol is only one part of place-based deterrence (Sherman 1990), we can still treat τ_1 as a close proxy for the duration of police patrol (Koper 1995).

Once place-based deterrence ends, a site is then open to be discovered and established as suitable for crime. Reported crime and disorder events presumably follow shortly after a site becomes favorable to crime. Crime site discovery happens at a rate λ_{02} . The waiting time between discovery events is τ_2 which has a probability density function $f(\tau_2) = \lambda_{02}e^{-\lambda_{02}\tau_2}$. However, this waiting time applies equally to all open or empty locations, including sites empty after a previous period of deterrence as well as a previous period of crime site suitability. However, since Poisson processes are separable (Bertsekas and Tsitsiklis 2008), we can use the known proportions of empty space arising from different prior states to partition the overall discovery rate into separate processes. Specifically, $\lambda_{02} = \pi_{10}\lambda_{02} + \pi_{20}\lambda_{02}$, where $\pi_{10}\lambda_{02}$ is the rate constant for the discovery of suitable crime sites following a prior period of deterrence and $\pi_{20}\lambda_{02}$ the rate constant following a prior period of crime site suitability. To simplify the notation, let $\tilde{\lambda}_{02} = \pi_{10}\lambda_{02}$ be the rate constant for the portion of space where crime site discovery follows the end of deterrence. Let $\tilde{\tau}_2$ be the corresponding waiting time, which has a probability density $f(\tilde{\tau}_2) = \tilde{\lambda}_{02}e^{-\tilde{\lambda}_{02}\tilde{\tau}_2}$.

For all practical purposes, $\tilde{\tau}_2$ measures the period of residual deterrence. That is, $\tilde{\tau}_2$ is the time between the end of initial deterrence at τ_1 and the time when crime and disorder re-emerge (Sherman 1990; Koper 1995). Figure 5A shows the two probability density functions together with the function for τ_2 , which is the waiting time for the discovery of any site suitable for crime. Recall that, at equilibrium, the rate at which initial deterrence is established is the same as the rate at which it ends, ensuring that the proportion of space subject to initial deterrence remains constant. Thus, for the parameter regime illustrated in Fig. 2B, it is the case that the turnover in suitable crime sites is faster than the turnover in initial deterrence (i.e., $E[\tau_2] < E[\tau_1]$). This suggests that in an ‘abundant crime world’ offenders take advantage of open space by moving

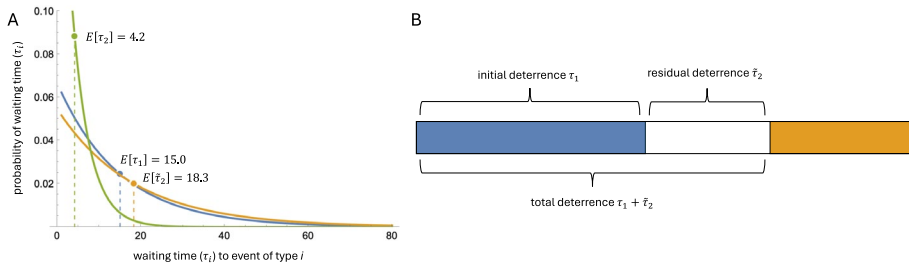


Fig. 5 Waiting time probability density functions and their relationship to stages of deterrence. A. Waiting times for the end of active deterrence and the discovery of suitable crime sites for the system shown in Fig. 2B. Waiting time τ_2 for the discovery of any open site for suitable for crime is shown in green. Waiting time $\hat{\tau}_2$ only for those sites following initial deterrence shown in orange. B. The duration of initial deterrence is measured by τ_1 (blue). Residual deterrence is measured by $\hat{\tau}_2$ (white). Total deterrence covers the period from the onset of initial deterrence and the re-emergence of crime and disorder (orange) and is given by $\tau_1 + \hat{\tau}_2$

faster than police, a key point matching the conclusion reached by Tilman (1994). However, if we restrict the analysis to only that portion of open space that emerged after the end of initial deterrence π_{10} , then the waiting time to discover a suitable crime site is actually longer than the waiting time for the end of initial deterrence (that is, $E[\hat{\tau}_2] > E[\tau_1]$) for the parameter regime in Fig. 2B. For sites going through this sequence of state transitions, it appears that crime site discovery is slower than the turnover of deterrence. Since initial deterrence is relatively rare in the environment (see Fig. 2B), opportunities to discover suitable crime sites after the end of initial deterrence are also rare (see Fig. 4). The time it takes to randomly discover such sites increases as the density of initial deterrence decreases (Holling 1959).

Now consider how τ_1 and $\hat{\tau}_2$ are related to the time a place is crime-free. There are two possible measures to consider (Fig. 5B). The first is what I call the *total deterrence* time. This encompasses the period of initial *and* the period of residual deterrence and is captured in the model by $\tau_1 + \hat{\tau}_2$. We can sum the waiting times because we are dealing only with the part of empty space $\pi_{10}(1 - p_1^* - p_2^*)$ that was previously under initial deterrence. The total waiting is the minimum time we should expect to see before crime and disorder emerge in a location, beginning from the onset of a bout of place-based policing. The second measure is the period of residual deterrence as defined by Sherman (1990) and Koper (1995). Residual deterrence is here captured by $\hat{\tau}_2$ alone. This is the time it takes for crime and disorder to emerge after the end of initial deterrence.

Figure 6 plots histograms of simulated waiting times⁵ until crime and disorder emerge in a large number of places, using rates λ_{10} and λ_{02} and parameters from Fig. 2B. The probability density for total deterrence waiting times shows an internal mode and a right skew (Fig. 6A). On aggregate, short waiting times for crime and disorder to emerge occur with low probability. The probability then rises quickly to what we might call a ‘buffered waiting time,’ by analogy with journey-to-crime distributions (Brantingham and Brantingham 1981; Rossmo and Wheeler 2024). Long waiting times occur, but with decreasing frequency. The theoretical probability density function in Fig. 6A is a mixture distribution computed

⁵To avoid confusion, I use *waiting time* to refer to the probability of an event such as the end of initial deterrence first happens at time τ . The waiting time is characterized by a probability density function $f(\tau)$. I use *survival time* to refer to the cumulative probability that a site remains crime-free at time τ given some form of prior deterrence.

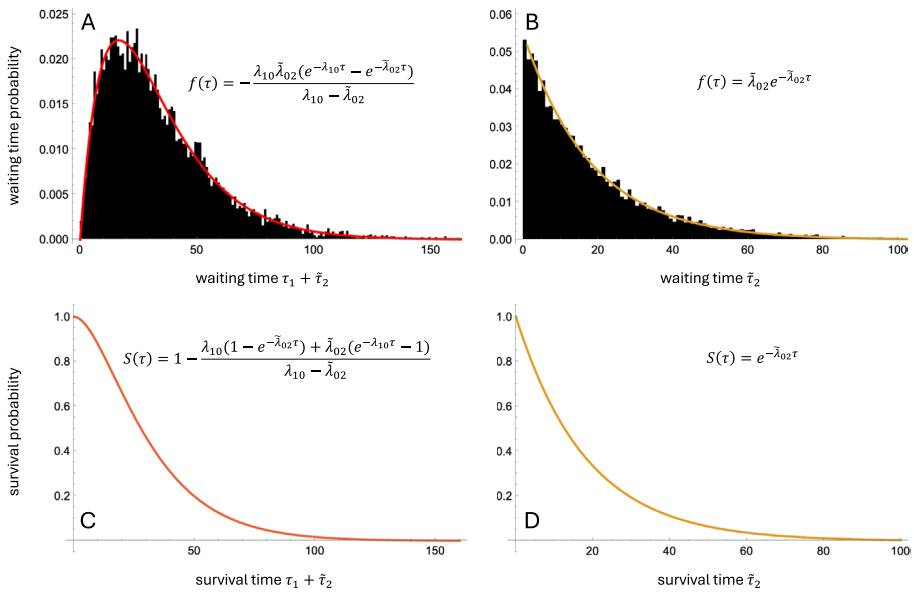


Fig. 6 Features of deterrence decay. A and B. Histograms of simulated waiting times and the theoretical probability density functions for the waiting time for crime site (re)discovery. C and D. Survival probability distributions. A and C. Total deterrence case from measured from the onset of deterrence. B and D. Residual deterrence case measured from the end of initial deterrence. Values of λ_{10} and $\tilde{\lambda}_{02}$ correspond to parameters in Fig. 2B

as the convolution of the two waiting time distributions for τ_1 and $\tilde{\tau}_2$. If $\lambda_{10} = \tilde{\lambda}_{02}$, then the convolution operation yields a gamma distribution with the common rate λ and shape parameter 2. If $\lambda_{10} \neq \tilde{\lambda}_{02}$ then the convolution yields a “gamma-like” distribution given as:

$$f(\tau) = -\lambda_{10}\tilde{\lambda}_{02} \frac{(e^{-\tau\lambda_{10}} - e^{-\tau\tilde{\lambda}_{02}})}{\lambda_{10} - \tilde{\lambda}_{02}} \quad (7)$$

In contrast, the waiting time for the emergence of crime and disorder given *only* residual deterrence is exponentially distributed (Fig. 6B) and follows exactly the probability density function shown in Fig. 5A for $\tilde{\tau}_2$. The theoretical probability density function for residual deterrence alone is $f(\tau) = \tilde{\lambda}_{02}e^{-\tilde{\lambda}_{02}\tau}$. After a period of initial deterrence, during which crime is suppressed, we should observe very short waiting times until the re-emergence of crime and disorder. The probability of observing longer waiting times decreases rapidly.

The corresponding survival functions $S(\tau)$ can be directly derived from the above equations. Here, the survival function gives the probability that a place will be crime-free up to a given time. In general $S(\tau) = 1 - \int_0^t f(\tau)dt$, where $f(\tau)$ is the (failure) waiting time density function of common notation. For the residual deterrence case, the survival function is simply $S(\tau) = e^{-\lambda_{02}\tau}$. For the total deterrence case, Eq. 7 integrates to:

$$S(\tau) = 1 - \frac{\lambda_{10}(1 - e^{-\tilde{\lambda}_{02}\tau}) + \tilde{\lambda}_{02}(e^{-\lambda_{10}\tau} - 1)}{\lambda_{10} - \tilde{\lambda}_{02}} \quad (8)$$

Figure 6C shows that the survival time under total deterrence starts slowly and then accelerates to an exponential form with increasing time since the onset of place-based policing. Figure 6D shows that the survival time under residual deterrence is exponential from the outset. These functional forms have important implications for optimal planning of police patrol that I take up in the Discussion.

Finally, I turn to the core elements of the Koper Curve (Koper 1995) which posits that longer place-based police patrols (up to a point) produce crime reduction benefits. Here, the expectation is that extending place-based patrol time will extend the waiting time until crime and disorder emerge, but longer patrols beyond should yield diminishing returns. Figure 7 shows the relationship between the two different measures of waiting time and the duration of the patrol as given by the present model. The relationship between the total deterrence waiting time $\tau_1 + \tilde{\tau}_2$ and patrol duration τ_1 is understandably linear (Fig. 7A). Patrol duration, which is equivalent to the period of initial deterrence in the model, sets a floor for total deterrence waiting time. Residual deterrence then adds an exponentially distributed random waiting time to this floor. The total deterrence waiting time expected from theory for any value of patrol duration τ_1 is simply $\tau_1 + 1/\tilde{\lambda}_{02}$. The functional relationship is a straight line with an intercept equal to $1/\tilde{\lambda}_{02}$ and a slope of one. In practice, the slope of the relationship estimated from data can be larger or smaller than the theoretically expected value, though the estimate will almost certainly include the expected slope in the 95% confidence interval. For example, in Fig. 7A, which shows 10^4 simulated observations, a linear model fit of the observed data produced an estimated slope of 1.001 with standard error 0.02, encompassing the predicted slope of 1. Estimated slopes less than 1 are also possible when fitting to observational data. Thus, the total deterrence mean waiting time can appear to be marginally shorter or longer at long patrol durations compared to short durations. Moreover, the deviation from the theoretical expectation can appear quite large for smaller observational sample sizes due to estimation error. However, the slope is generally not meaningfully different from that predicted by theory.

The model predicts no relationship between patrol duration τ_1 and residual deterrence $\tilde{\tau}_2$. Once the period of initial deterrence has ended, no matter how long, there is an exponentially distributed amount of time until crime and disorder can resume. Thus, the expected waiting time based on theory is a constant $1/\tilde{\lambda}_{02}$ for all values of τ_1 . Here, as well, models fit to observational data will tend to deviate from the theoretically expected

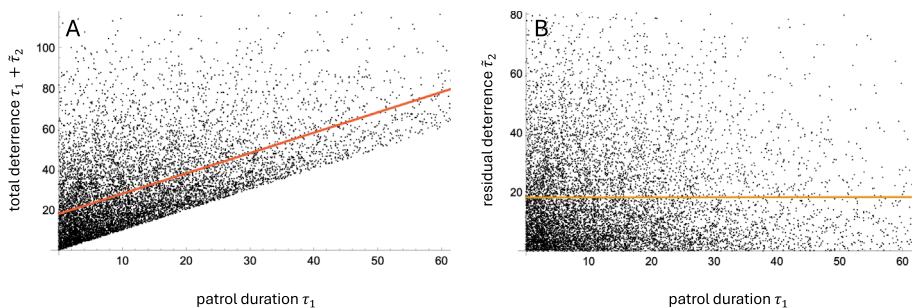


Fig. 7 Plot of deterrence against patrol duration for two cases. A. Total deterrence is the sum of the initial deterrence tied to patrol duration and the time to discover a suitable crime site after initial deterrence ends. B. Residual deterrence includes only the time to discover a suitable crime site after initial deterrence ends

pattern, but not in a meaningful way. For example, in Fig. 7B, the estimated slope is 0.002, but encompasses the expected slope of 0. A slightly negative non-significant slope is also possible when fitting to observational data.

Discussion

The model presented above produces two broad theoretical observations. First, in the face of place-based deterrence, there are specific conditions that can produce non-zero proportions of suitable crime sites at equilibrium. These conditions boil down to an observation that place-based policing necessarily produces a “patchy” distribution of deterrence and that crime persists if offenders are able to find and exploit open patches. This point is explored more fully by Tilman (1994) who was motivated by the case of plant species competing for space.⁶

Second, the waiting time for crime and disorder to emerge and the crime-free survival function might follow one of two different distributions depending on how the waiting time is measured. If the focus is so-called total deterrence, which includes the time since the onset of policing to the next event of crime and disorder occurs, then the probability density function for the waiting time is “gamma-like” with an internal mode and long tail. The total deterrence survival function changes slowly at first and then accelerates. If the focus is strictly on residual deterrence, then both the waiting time and survival function are exponential in form. Both the total and residual deterrence cases reflect particular forms of deterrence decay derived from theory. When plotted against the period of initial deterrence (i.e., patrol duration) to match the idea behind the Koper Curve, total deterrence waiting times increase with initial deterrence. However, this is entirely expected since the total deterrence survival time is defined as the sum of initial and residual deterrence survival time. When residual deterrence waiting times are plotted against the period of initial deterrence, there is no statistical relationship. The expected waiting time is constant for all values of patrol duration. Neither measure shows the dependency on the period of initial deterrence suggested by Koper (1995).

The practical implications of this last result are important to consider before turning to possible explanations for the discrepancy. The result raises interesting questions about the value of periodically reinforcing place-based interventions. To see why this is an issue, start with the residual deterrence survival function $S(\tau) = e^{-\hat{\lambda}_{02}\tau}$, which produces a survival probability of $S(0) = 1$ at the point where police leave a place and declines exponentially thereafter. Using the value of $\hat{\lambda}_{02} = 0.0547$ (rounded for presentation) underlying the dynamics in Fig. 1B, the probability of surviving 20 time units (assumed to be minutes) crime-free after police leave is $S(20) = 0.3351$. Imagine now that the police perform a maintenance visit to the place after 15 minutes, rather than the full 20 minutes, to “reset” the survival function to $S(0) = 1$. They spend a few minutes engaged place-based activities, and then leave again. To assess whether this is a beneficial strategy, we need to know if the probability of surviving crime-free for 15 minutes after the intervention *and* 5 more minutes

⁶For those skeptical of the use of plant ecology as inspiration for the study of human systems it is worth re-reading founding works of Chicago School Sociology. Park, Burgess, and McKenzie (1925) made liberal use of plant ecology in describing the structure and function of cities and found plenty of justification for their approach.

after the second—a total of 20 minutes of crime-free residual deterrence interrupted by a bout place-based maintenance—yields a greater probability of survival than just leaving the place alone for 20 minutes. We compute $S(15)S(5) = 0.4404 \times 0.7608 = 0.3351$, which is identical to $S(20) = 0.3351$. The unfortunate conclusion is that there is no benefit to optimizing place-based policing to reset deterrence conditions *if* residual deterrence survival times are exponentially distributed. There *appears* to be a benefit to periodic maintenance if the measure of interest is total deterrence (i.e., combining initial and residual deterrence). For example, using Eq. 8 and the values of $\lambda_{10} = 0.0667$ and $\lambda_{02} = 0.0547$ underlying Fig. 1B, then $S(20) = 0.6608$ while $S(15)S(5) = 0.7709 \times 0.8927 = 0.7422$, a slight improvement in the probability of survival. Appearances can be deceiving, however. Recall that the survival function in this case includes both the suppression effects of initial deterrence and the waiting time of residual deterrence for offenders to discover sites after police have left. Some fraction of the total survival time is initial deterrence, and thus the higher survival probability for the strategy with maintenance comes from simply adding more initial deterrence, not from any residual benefit.

The modeling results discussed above are interesting but they do not mean that the ideas behind the Koper Curve are wrong. As alluded to in the introduction to this paper, the discrepancy between deterrence decay as modeled here and the results of Koper reflect both different methods and different assumptions. Here, the results are derived from first principles and produce qualitative expectations. That is, although the model is mathematically rigorous and points to the family of survival distributions we should expect to see if the model assumptions are correct, they do not tell us exactly what the distribution parameters should be. The model parameters were chosen to produce a low density of police deterrence at equilibrium and deterrence bouts lasting 15 time units on average, with the intention that the units be consistent with the Koper Curve. However, calibration of abstract mathematical models is a challenging problem as noted by other criminologists (Nagin et al. 2015; Short et al. 2010). Thus, the goal of theory here is to think about mechanisms and process, rather than to test any specific prediction.

The differences in assumptions are also important. The theory invoked by Sherman (1990), Koper (1995), and others (Nagin et al. 2015; Loughran et al. 2011) focuses on the internal decision-making process of offenders and implies situations where offenders are continuously monitoring locations subject to policing. Deterrence decay in that framework is largely an internal cognitive process, although in Nagin et al. (2015) the risk that offenders are willing to bear is related to the characteristics of criminal opportunities. In contrast, the cognitive processes of the offender are invoked in the present model only indirectly in that they could be the basis for the competitive superiority of initial deterrence over crime site suitability. Thus, a reasonable interpretation of the present model is that a location with initial deterrence is absolutely unfavorable to crime because of the risk preferences of all offenders who encounter that place. Recognize, however, that the model does not suggest that *deterrence decay* operates in the heads of offenders. Rather, in the model, if police are present in a place, then deterrence is absolute. If police are absent, there is no deterrence. Offender risk preference in the model is a step function.

Nevertheless, a type of deterrence decay is observed in the model. It operates on the aggregate scale. Initial deterrence decay is seen in the frequency distribution of patrol durations across all police patrols. The decay function should be exponential with a rate corresponding to the mean duration of the patrol. A large number of sites experience short patrols and a few sites experience long patrols. This is a reasonable way to understand

what happens with both routine and hotspot policing, with some directive to remain in places of interest for a certain amount of time (Telep et al. 2014). Specifically, if calls-for-service are a stationary (constant) Poisson arrival process, then discretionary police patrols should end following that stationary Poisson process when prompted to act by incoming calls. The result will be patrol durations that are exponentially distributed at the aggregate scale.

A similar observation concerns the functional form of residual deterrence decay. Within the model, this is observed in the distribution of waiting times until a site is discovered and established as suitable for crime after initial deterrence as ended. Here, too, residual deterrence is exponentially distributed at the aggregate scale. One interpretation of the underlying process is that offenders have limited time (or attention) to monitor a collection of places. Whether or not police have left a place requires motivated offenders to drop what they are doing (e.g., non-criminal activity) and dedicate time and energy to looking for opportunities. If the time-creating interruptions arise from a stationary Poisson process, then the waiting time to onset of search should be exponentially distributed. If the rate at which suitable places are encountered is constant, then the waiting time to discovery will also be exponentially distributed according to a Poisson process.

The ecological mechanisms for initial and residual deterrence suggested here are not mutually exclusive with the psychologically-grounded mechanisms suggested by Sherman (1990) and central to the Koper Curve. Rather, both mechanisms might operate in parallel, with one or the other being more important depending on the situation. For example, certain preferred places on the landscape, such as public drug markets, cannot be easily replicated elsewhere. Offenders, therefore, might monitor place-based interventions in preferred places more closely to determine when it is safe to resume illicit activities. In such situations, we might expect patterns of residual (and initial) deterrence to exhibit the features described by Sherman and Koper. The uncertainty surrounding the chance of being caught may be the controlling factor in returning to business as usual in preferred places. However, many other types of crime and disorder, such as car theft or public antisocial behavior, are more opportunistic in nature. The relationship between crime and particular places is more a matter of convenience (or chance) than preference for these event types. Offenders may not monitor place-based interventions in convenient places since it is easy to pass those opportunities blocked by police presence. In these situations, we might expect patterns of residual (and initial) deterrence to exhibit the features described by the present model. The time it takes to rediscover chance opportunities may be the controlling factor in returning to business as usual in convenient places. In general, crime landscapes almost certainly represent a mixture of preferred and convenient places. The observed character of place-based deterrence likely depends how place-based interventions sample that mixture. More research will be needed to explore these possibilities.

There are a number of limitations to the present model that are worth noting. First, the competitive dominance assumption central to the model is a useful first approximation of place-based interactions between police and offenders. There are certainly numerous empirical situations where competitive dominance is unlikely to hold. For example, police may be able to suppress crime and disorder in outdoor locations, where direct interactions between police and motivated offenders or suitable targets are possible, but not crimes

committed behind closed doors. It may be possible to relax competitive dominance assumptions using more complex compartmental models or a kinetic equations framework that treats deterrence as a probability distribution over places (Bellomo et al. 2015; Sisk et al. 2022). These approaches could allow for variable place-based interventions (e.g., shot drive-bys vs. longer interventions) as well as responses that differ by crime type. I leave this for future work.

Second, place-based policing is premised on the recognition that spatial processes are central to crime patterns and that those patterns can be leveraged to allocate police time and energy. Yet, here, space is treated implicitly, which induces some unrealistic behavior. For instance, distance does not matter at all in the model. From the perspective of the individual police officer or offender in the model, a site that nominally opens up on the other side of the “world” is no more difficult to access than one that opens up right next door. This is certainly not true in real-world settings. Similarly, the model assumes that it is just as easy or difficult to establish deterrence or crime site suitability in an open site that has never been visited before as it is in a site that has been visited many times before. There is no environmental memory in the system. In real-world settings, a site highly favorable to crime before a bout of crime suppression may be more likely to resume this status after initial and residual deterrence ends, whereas a site moderately favorable for crime would resume that status. Age-structured models could be adapted for this purpose (Sanchez et al. 2019). A more direct approach would be to use spatially explicit models based on partial differential equations that include distance and spatial memory of conditions (Calatayud et al. 2023; Lloyd et al. 2016; Rodriguez et al. 2021; Short et al. 2010). Spatially explicit models might produce markedly different results.

Third, the model developed here operates at a scale above the discrete, moment-to-moment churn of policing and crime at geographic micro-places such as street segments or sub-neighborhood-sized areal units. It describes the average or “mean-field” behavior of the system. At the street level, policing and crime do not appear to be equilibrium processes (Mohler et al. 2017). Yet, when we abstract away from the many stochastic events at the street level to aggregate measures such as the proportion of space given over to initial deterrence or suitable for crime, we expect to see much greater spatial and temporal pattern stability (Mohler et al. 2011). Stable aggregate patterns in policing and crime suggest that mean-field models are appropriate and warrant the examination of their equilibrium properties. Moreover, mean-field models can be derived from their stochastic counterparts used to describe the fine-grained behavior of street-level systems (Higham 2001; Short et al. 2008).

Finally, it is true that one could arrive at the final results of the paper by simply assuming that the two rates of primary interest—the end of initial deterrence and discovery of crime site suitability—are Poisson and then immediately proceed to discussing the associated waiting time and survival distributions. The unique contribution here lies in providing a mechanistic or ecological rationale, with a corresponding mathematical model, for Poisson rates. The model is a gross simplification, but nevertheless seeks to capture key features of how the real-world works. The number of unique rates to be tracked and the relative (and absolute) values of those rates are a product of the ideas instantiated in the model. The so-called Gillespie method for translating continuum mathematical models into discrete simulations also plays a role in connecting the mathematical theory suggested to measures

that could be explored empirically (Gillespie 1977). The Gillespie method is more than just a convenience. It has proven to be a quantitatively accurate way of moving between discrete and continuous systems, at least in the domains in which it is most commonly used, such as chemistry, epidemiology, and theoretical ecology. It is a good starting point for the current domain, but will require substantial future work to verify.

Supplementary information *Mathematica* and *Python* notebooks are included to reproduce the findings in this paper.

Appendix A: Density-independent Models

Equations 1 and 2 assume density dependence in growth rates. The assumption implies that it becomes easier to establish initial deterrence and discover suitable crime sites as more of space is given over to these respective states. For example, when initial deterrence is rare, $c_1 p_1' (1 - p_1') > c_1 p_1 (1 - p_1)$ for $p_1' > p_1$.

Alternative assumptions that remove these density dependence in growth lead to qualitatively similar outcomes. Assume here that the fundamental rates at which initial deterrence and crime site suitability are established are constant. That is, they do not benefit from other places in the environment already being in these states. This simplified setting can be expressed mathematically as (Gotelli 1998):

$$\frac{dp_1}{dt} = c_1(1 - p_1) - m_1 p_1 \quad (\text{A1})$$

$$\frac{dp_2}{dt} = c_2(1 - p_1 - p_2) - m_2 p_2 - c_1 p_2 \quad (\text{A2})$$

Note in Eq. A1 the constant supply of initial deterrence c_1 that is only curtailed as space fills up $(1 - p_1)$. Similarly, in Eq. A2, there is constant supply of offender interest c_2 that is only curtailed as space is filled up with initial deterrence and existing suitable crime sites $(1 - p_1 - p_2)$. The rates at which initial deterrence $m_1 p_1$ and crime site suitability $m_2 p_2$ end are proportionally dependent since such states have to first exist in order for them to cease to exist. The rate of crime site suppression is dependent only upon the constant supply of initial deterrence and the proportion of space given over to crime $c_1 p_2$. In ecological settings, Eq. A1 is called a “propagule rain” model.

In spite of the difference between the main model and simplified model given by Eqs. A1 and A2, the dynamics are qualitatively similar (Fig. 8). The simplified model thus yields similar state transition rates and waiting times that can be interpreted in the same way as in the main model. For example, in the simplified model, the rate at which initial deterrence ends is still $m_1 p_1$ while the rate at which crime sites are discovered is $c_2(1 - p_1^* - p_2^*)$, which produces a qualitatively similar waiting time distribution after restricting to that portion of empty space that emerged from initial deterrence (Fig. 8B).

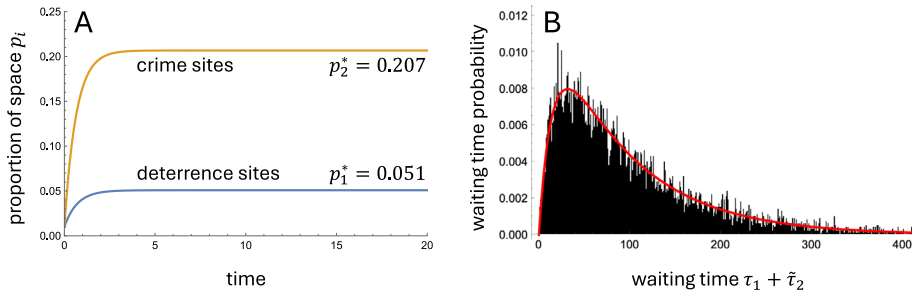


Fig. 8 A “propagule rain” model where the supply of initial deterrence and crime site interest are constant. A. Temporal trajectory for the proportion of sites with initial deterrence p_1 and suitable for crime p_2 . Compare with Fig. 2B. B. Histogram of simulated total deterrence waiting times and the theoretical probability density functions for the waiting time for crime site rediscovery. Compare with Fig. 6A. The computation takes into account the partitioning of space as shown in Fig. 4. Parameter values $c_1 = 0.071$, $m_1 = 1.333$, $c_2 = 0.321$, $m_2 = 1.083$ were chosen to produce equilibrium proportions similar to Fig. 2B and a mean waiting time to the end of initial deterrence of 15 time units

Appendix B: Full Branching Structure

Figure 4B in the main text focuses on the three-state sequences that start with places subject to either initial deterrence or suitable for crime. Figure 9 shows the complete branching structure for all of the ways in which three-state sequences can be assembled. The landscape frequency of each intermediate node type is given as $p_j^* \pi_{ij}$ and each leaf node type as $p_k^* \pi_{ij} \pi_{jk}$, where i is the initial state, j is the intermediate state, and k is the final state. For example, the proportion of space at an instantaneous moment that followed the sequence from empty to suitable for crime to initial deterrence (i.e., crime suppression) is computed

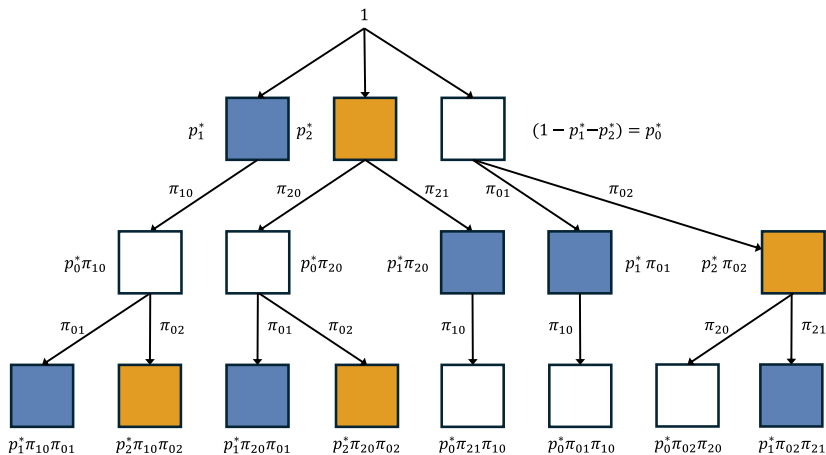


Fig. 9 The full branching structure for places that experienced a sequence of exactly three states (two transitions) at some instantaneous point in time. The terminal leaf nodes give the expected proportion of places space in state k that followed the sequence of states i to j to k . Coloring is the same as in Fig. 4B

as $p_1^* \pi_{02} \pi_{21}$. Note that the eight terms at the leaf nodes in Fig. 9 sum to 1, indicating that *all locations* in the environment, surveyed at a single point in time, must have followed one of the listed trajectories.

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Data Availability Not Applicable.

Code Availability *Mathematica* and *Python* notebooks are included to reproduce the findings in this paper.

Materials Availability Not Applicable.

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